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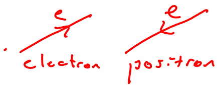
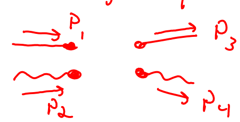
Feynman Rules for QED

1. Draw diagram and label matter lines w/ arrows to distinguish particles/anti, e.g.

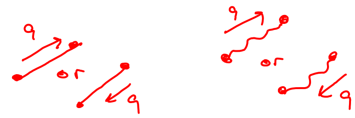


label momenta: a) external

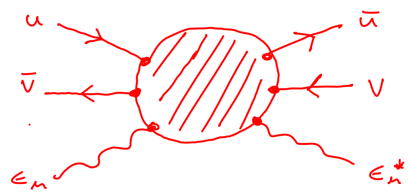


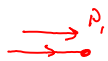
(along T)

b) internal



2. Each <u>external</u> line gets a factor according to:



Number factors w/ momenta, e.g.  $\Rightarrow u(1)$

Note: Each factor is a 4spinor or 4vector.

3. Each vertex gets a factor $i g_e \gamma^\mu$ $\left(g_e = e\sqrt{\frac{4\pi}{\hbar c}}\right)$ Note: Overall spin matrix.

4. Each <u>internal</u> line gets a factor: Electrons/Positrons $\frac{i(\gamma^\mu q_\mu + mc)}{q^2 - m^2c^2}$ Overall spin matrix.

Photon $-\frac{i g_{\mu\nu}}{q^2}$ Overall tensor (μ,ν match vertex indices)

5. Conserve 4momentum at each vertex w/ $(2\pi)^4 \delta^4(p_{in} + q_{in} - p_{out} - q_{out})$

6. Integrate over internal momenta w/ $\int \frac{d^4q}{(2\pi)^4}$ for each q.

7. Cancel overall $(2\pi)^4 \delta^4(p_{in} - p_{out})$ and $\times i$ to get $\mathcal{M}$.

8. Antisymmetrize between diagrams related by switching 2 incoming electrons/positrons, 2 outgoing electrons/positrons or one incoming electron/positron with one outgoing positron/electron.

9. To get the order right for spinor elements (since we suppress indices) make "spinor sandwiches from matter lines" by starting w/ an outgoing matter particle and tracing it back along pure matter lines writing polarization and vertex factors as needed. The photon factors are less tricky since index notation gets them right.

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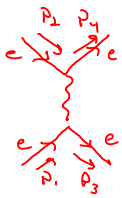
Example:

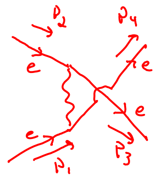
$e + e \to e + e$



$$\int \bar{u}(3)\, i g_e \gamma^\mu u(1)\, \bar{u}(4)\, i g_e \gamma^\nu u(2) \left(\frac{-i g_{\mu\nu}}{q^2}\right) (2\pi)^4 \delta^4(p_1 - q - p_3)\, (2\pi)^4 \delta^4(p_2 + q - p_4) \frac{d^4 q}{(2\pi)^4}$$

$$\bar{u}(3)\, i g_e \gamma^\mu u(1)\, \bar{u}(4)\, i g_e \gamma^\nu u(2) \left(\frac{-i g_{\mu\nu}}{(p_4 - p_2)^2}\right) (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4)$$

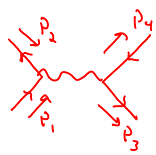
$$\Rightarrow M_1 = \bar{u}(3)\, i g_e \gamma^\mu u(1)\, \bar{u}(4)\, i g_e \gamma^\nu u(2) \left(\frac{-i g_{\mu\nu}}{(p_4 - p_2)^2}\right) i$$



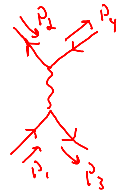
$M_2 = M_1 (3 \leftrightarrow 4)$

By step 8 : $M = M_1 - M_2$

Consider $e^+ + e^- \to e^+ + e^-$:

1. 

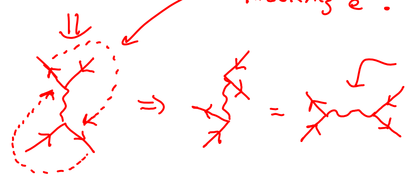
M_1

2. 

M_2

$M \overset{?}{=} M_1 \pm M_2$

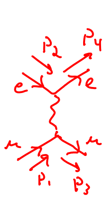
switch one outgoing e^- with one incoming e^+.



this is the same as in case 1.

So $M = M_1 - M_2$

Consider $e + \mu \to e + \mu$



Note: No contribution from 

$$\Rightarrow \quad M = \bar{u}(3) i g_e \gamma^\mu u(1)\, \bar{u}(4) i g_e \gamma^\nu u(2) \left(\frac{-i g_{\mu\nu}}{(p_4-p_2)^2}\right) i$$

Once we have M, we can evaluate it given the following:

$u(i) = \alpha_i u^{(1)}(p_i) + \beta_i u^{(2)}(p_i) \qquad i = 1,2,3,4$

However in most experiments we average over $\alpha_1, \beta_1, \alpha_2, \beta_2$ and sum over $\alpha_3, \beta_3, \alpha_4, \beta_4$.

To do the avg/sum we first note that M contains terms of the form:

$$M = \cdots \times \underbrace{\bar{u}(a)\, \Gamma_1\, u(b)}_{\text{Spinor sandwhich}}$$

some combination of spin matrices

label to distinguish from other cases

Then:

$$|M|^2 = \cdots \bar{u}(a)\, \Gamma_1\, u(b) \left[\underbrace{\bar{u}(a)\, \Gamma_2\, u(b)}\right]^* \cdots$$

Note: We give this a different label. In particular if Γ_1 carries spacetime indices μ, ν then Γ_2 will need different indices α, λ to avoid confusing contractions.

This thing is a number in spin space so we can freely transpose it to form $\dagger$.

$$= \cdots \bar{u}(a)\, \Gamma_1\, u(b) \left[\underbrace{\bar{u}(a)\, \Gamma_2\, u(b)}\right]^\dagger$$

$$\begin{aligned} u(b)^\dagger \Gamma_2^\dagger \bar{u}(a)^\dagger &= u(b)^\dagger \Gamma_2^\dagger \left(u(a)^\dagger \gamma^0\right)^\dagger \\ &= u(b)^\dagger \Gamma_2^\dagger \gamma^{0\dagger} u(a) \end{aligned}$$

insert $\gamma^0\gamma^0 = I$ and use $\gamma^0 = \gamma^{0\dagger}$

$$\begin{aligned} &= u(b)^\dagger \gamma^0 \gamma^0 \Gamma_2^\dagger \gamma^0 u(a) \\ &= \bar{u}(b)\, \bar{\Gamma}_2\, u(a) \end{aligned}$$

$\gamma^0 \Gamma_2^\dagger \gamma^0$

$$= \cdots \bar{u}(a)\, \Gamma_1\, u(b)\, \bar{u}(b)\, \bar{\Gamma}_2\, u(a)$$

Now to avg/sum the first step will be to sum so we can use $\sum_s u^{(s)} \bar{u}^{(s)} = \gamma^\mu p_\mu + mc = \not{p} + mc$

$$\sum_{s_b} |M|^2 = \cdots \underbrace{\bar{u}(a)}_{\text{Row}} \underbrace{\Gamma_1 (\not{p}_b + m_b c)\, \bar{\Gamma}_2}_{\text{Matrix}} \underbrace{u(a)}_{\text{Column}} \cdots$$

Now we can use that $RMC = \# = Tr(MCR)$

$$\sum_{s_b} |M|^2 = \cdots Tr\left[\Gamma_1 (\not{p}_b + m_b c)\, \bar{\Gamma}_2\, u(a) \bar{u}(a)\right] \cdots$$

Now sum over s_a to get:

$$\sum_{s_a, s_b} |M|^2 = \cdots Tr\left[\Gamma_1 (\not{p}_b + m_b c)\, \bar{\Gamma}_2 (\not{p}_a + m_a c)\right] \cdots$$

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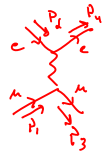
If at some point we had summed over $v\bar{v} \Rightarrow \not{p} - mc$.

Note: There are no spinors left in the expression! Only p_μ's and γs!

$$\boxed{\text{To impose sum over } s_a, s_b \text{ replace } \bar{u}(a)\Gamma_1 u(b)[\bar{u}(a)\Gamma_2 u(b)]^* \Rightarrow Tr[\Gamma_1(\not{p}_b + m_b c)\bar{\Gamma}_2(\not{p}_a + m_a c)]}$$

Let's put this result to work:

$e+\mu \rightarrow e+\mu$



$$M = -\frac{g_e^2}{(p_4-p_2)^2}\bar{u}(3)\gamma^\mu u(1)\bar{u}(4)\gamma^\nu u(2) g_{\mu\nu}$$

$$|M|^2 = \frac{g_e^4}{(p_4-p_2)^4}\bar{u}(3)\gamma^\mu u(1)\bar{u}(4)\gamma^\nu u(2) g_{\mu\nu}[\bar{u}(3)\gamma^\lambda u(1)\bar{u}(4)\gamma^\alpha u(2) g_{\lambda\alpha}]^*$$

$$Tr[\gamma^\mu(\not{p}_1 + m_1 c)\bar{\gamma}^\lambda(\not{p}_3 + m_3 c)]$$

$$Tr[\gamma^\nu(\not{p}_2 + m_2 c)\bar{\gamma}^\alpha(\not{p}_4 + m_4 c)]$$

2 incoming particles w/ 2 spin states each

$$\langle |M|^2 \rangle = \frac{1}{4}\frac{g_e^4}{(p_4-p_2)^4} Tr[\gamma^\mu(\not{p}_1 + m_1 c)\bar{\gamma}^\lambda(\not{p}_3 + m_3 c)]Tr[\gamma^\nu(\not{p}_2 + m_2 c)\bar{\gamma}^\alpha(\not{p}_4 + m_4 c)]g_{\mu\nu}g_{\lambda\alpha}$$